In the case of the hand-held digital voltmeter the dominant uncertainty contribution resulting from the finite resolution of the indication is $u_{\delta V_X}(E_X) = 0{,}029\ \text{V}$ whereas the total uncertainty contribution of the non-dominant terms is $u_R(E_X) = 0{,}0064\ \text{V}$. The relevant ratio is $u_R(E_X)/u_{\delta V_X}(E_X) = 0{,}22$. Thus the resulting distribution of values that can reasonably be attributed as errors of indications is essentially rectangular. The coverage probability for a rectangular distribution is linearly related to the expanded measurement uncertainty (a being the half-width of the rectangular distribution)

$$p = \frac{U}{a}\,. \qquad \text{(S9.7)}$$

Solving this relation for the expanded measurement uncertainty U and inserting the result together with the expression of the standard measurement uncertainty related to a rectangular distribution as given by eq. (3.8) of EAL-R2 finally gives the relation

$$k(p) = p\sqrt{3}\ . \qquad \text{(S9.8)}$$

For a coverage probability $p = 95\ \%$ applicable in the EA, the relevant coverage factor is thus $k = 1{,}65$.

S10 CALIBRATION OF A VERNIER CALLIPER

S10.1 A vernier calliper made of steel is calibrated against grade I gauge blocks of steel used as working standards. The measurement range of the calliper is 150 mm. The reading interval of the calliper is 0,05 mm (the main scale interval is 1 mm and the vernier scale interval 1/20 mm). Several gauge blocks with nominal lengths in the range 0,5 - 150 mm are used in the calibration. They are selected in such a way that the measurement points are spaced at nearly equal distances (e.g. at 0 mm, 50 mm, 100 mm, 150 mm) but give different values on the vernier scale (e.g. 0,0 mm, 0,3 mm, 0,6 mm, 0,9 mm). The example concerns the 150 mm calibration point for measurement of external dimensions. Before calibration several checks of the condition of the calliper are made. These include dependence of the result of measurement on the distance of the measured item from the beam (Abbe error), quality of the measuring faces of the jaws (flatness, parallelism, squareness), and function of the locking mechanism.

S10.2 The error of indication E_X of the calliper at the reference temperature $t_0 = 20°\,\text{C}$ is obtained from the relation:

$$E_X = l_{iX} - l_S + L_S \cdot \overline{\alpha} \cdot \Delta t + \delta l_{iX} + \delta l_M \qquad \text{(S10.1)}$$

where:

l_{iX}	-	indication of the calliper,
l_S	-	length of the actual gauge block,
L_S	-	nominal length of the actual gauge block,
$\overline{\alpha}$	-	average thermal expansion coefficient of the calliper and the gauge block,
Δt	-	difference in temperature between the calliper and the gauge block,
δl_{iX}	-	correction due to the finite resolution of the calliper,
δl_M	-	correction due to mechanical effects, such as applied measurement force, Abbe errors, flatness and parallelism errors of the measurement faces.

S10.3 **Working standards (l_S, L_S)**

The lengths of the reference gauge blocks used as working standards, together with their associated expanded uncertainty of measurement, are given in the calibration certificate. This certificate confirms that the gauge blocks comply with the requirements for grade I gauge blocks according to ISO 3650, i.e. that the central length of the gauge blocks coincides within ±0,8 µm with the nominal length. For the actual lengths of the gauge blocks their nominal lengths are used without correction, taking the tolerance limits as the upper and lower limits of the interval of variability.

S10.4 **Temperature (Δt, $\overline{\alpha}$)**

After an adequate stabilisation time, the temperatures of the calliper and the gauge block are equal within ±2 °C. The average thermal expansion coefficient is $11{,}5 \cdot 10^{-6}$ $°\text{C}^{-1}$. (The uncertainty in the average thermal expansion coefficient and in the difference of the thermal expansion coefficients has not been taken into account; its influence is considered negligible for the present case. Cf. EAL-R2-S1, example S4.)

S10.5 **Resolution of the calliper (δl_{iX})**

The scale interval of the vernier scale is 0,05 mm. Thus variations due to the finite resolution are estimated to have rectangular limits of ± 25 µm.

S10.6 **Mechanical effects (δl_M)**

These effects include the applied measurement force, the Abbe error and the play between the beam and the sliding jaw. Additional effects may be caused by the fact that the measuring faces of the jaws are not exactly flat, not parallel to each other and not perpendicular to the beam. To minimise effort, only the range of the total variation, equal to ±50 µm is considered.

S10.7 **Correlation**

None of the input quantities are considered to be correlated to any significant extent.

S10.8 Measurements (l_{iX})

The measurement is repeated several times without detecting any scatter in the observations. Thus uncertainty due to limited repeatability does not give a contribution. The result of measurement for the 150 mm gauge block is 150,10 mm.

S10.9 Uncertainty budget (δl_X)

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
l_{iX}	150,10 mm	-	-	-	-
l_S	150,00 m	0,46 µm	rectangular	-1,0	-0,46 µm
Δt	0	1,15 K	rectangular	1,7 µMk^{-1}	2,0 µm
δl_{iX}	0	15 µm	rectangular	1,0	15 µm
δl_M	0	29 µm	rectangular	1,0	29 µm
E_X	0,10 mm				33 µm

S10.10 Expanded uncertainty

The uncertainty of measurement associated with the result is clearly dominated by the combined effect of the measurement force and the finite resolution of the vernier. The final distribution is not normal but essentially trapezoidal with a ratio $\beta = 0{,}33$ of the half-width of the plateau region to the half-width of the variability interval. Therefore the method of effective degrees of freedom described in EAL-R2, Annex E is not applicable. The coverage factor $k = 1{,}83$ appropriate for this trapezoidal distribution of values is calculated from eq. (S10.10) of the mathematical note S10.13. Thus

$$U = k \cdot u(E_X) = 1{,}83 \cdot 0{,}033 \text{ mm} \cong 0{,}06 \text{ mm}$$

S10.11 Reported result

At 150 mm the error of indication of the calliper is (0,10 ± 0,06) mm.

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor $k = 1{,}83$ which has been derived from the assumed trapezoidal probability distribution for a coverage probability of 95 %.

S10.12 Additional remark

The method used for calculating the coverage factor is clearly related to the fact that uncertainty of measurement associated with the result is dominated by two influences: the mechanical effects and the finite resolution of the vernier scale. Thus the assumption of a normal distribution for the output quantity is not justified and the conditions of EAL-R2, paragraph 5.6 apply. In the sense that probabilities and probability densities in practice may only be determined to within 3 %– 5 %, the distribution is essentially trapezoidal, obtained by convolution of the two rectangular distributions associated with the dominant contributions. The half-widths of the base and the top of the resulting symmetrical trapezoid are 75 µm and 25 µm,