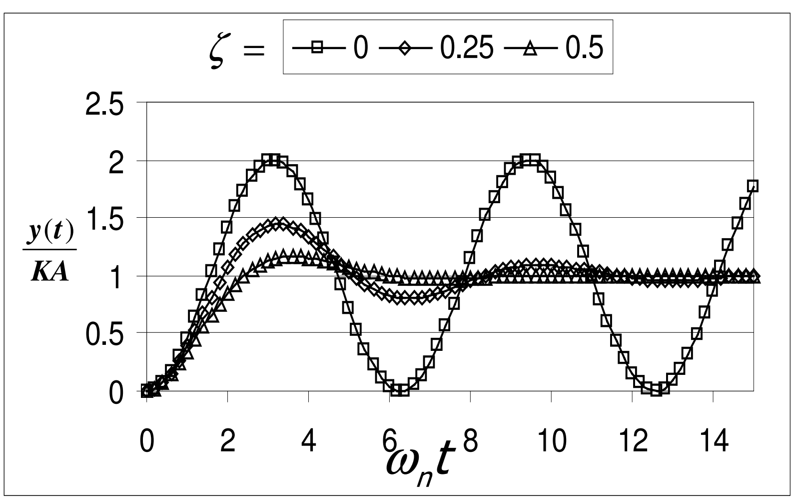
Second Order System

Note: ω_n is the natural frequency of the system, while ω is the frequency of the input signal

Step input - Time response

$$0 \le \zeta < 1$$

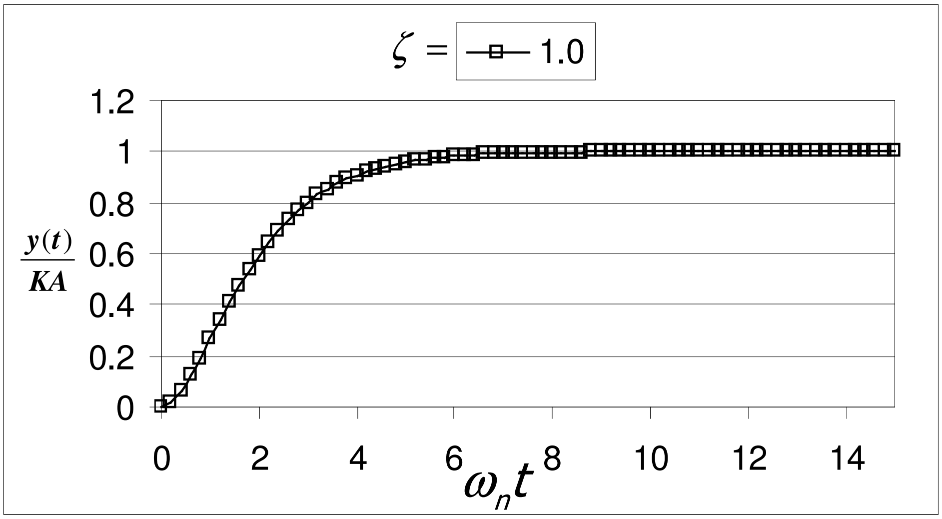
$$y(t) = KA - KAe^{-\zeta\omega_n t}\left[\frac{\zeta}{(1-\zeta^2)^{1/2}}\sin(\omega_n\sqrt{1-\zeta^2}\,t) + \cos(\omega_n\sqrt{1-\zeta^2}\,t)\right]$$



Step input - Time response

$\zeta = 1$

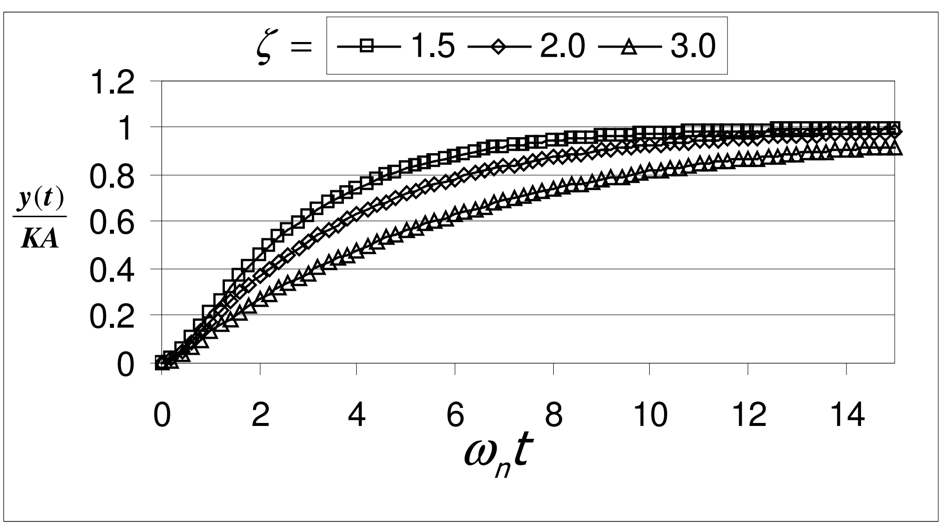
$$y(t) = KA - KA(1 + \omega_n t)e^{-\omega_n t}$$



Step input - Time response

$\zeta > 1$

$$y(t) = KA - KAe^{-\zeta\omega_n t}\left[\frac{\zeta}{(1-\zeta^2)^{1/2}}\sinh(\omega_n\sqrt{1-\zeta^2}t) + \cosh(\omega_n\sqrt{1-\zeta^2}t)\right]$$



Sine input: frequency response

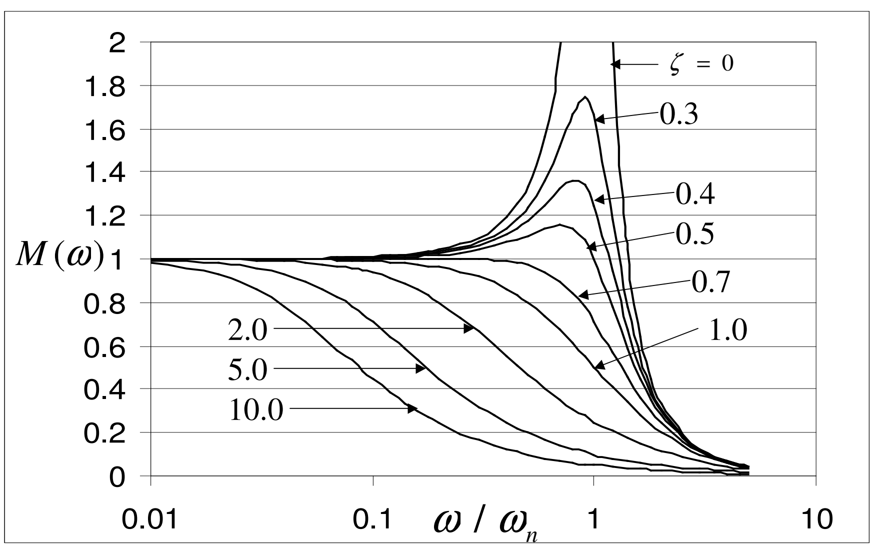
$$y(t) = y_h + \frac{KA\sin[\omega t + \phi(\omega)]}{\left\{\left[1-(\omega/\omega_n)^2\right]^2 + (2\zeta\omega/\omega_n)^2\right\}^{1/2}}$$

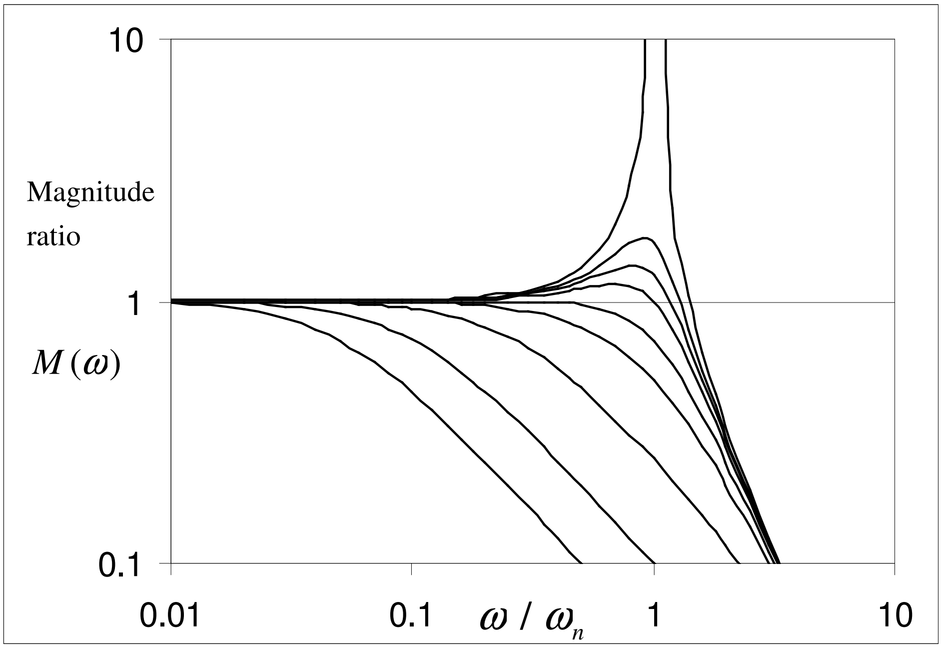
$$\phi(\omega) = \tan^{-1}\frac{-2\zeta\omega/\omega_n}{\left[1-(\omega/\omega_n)^2\right]}$$

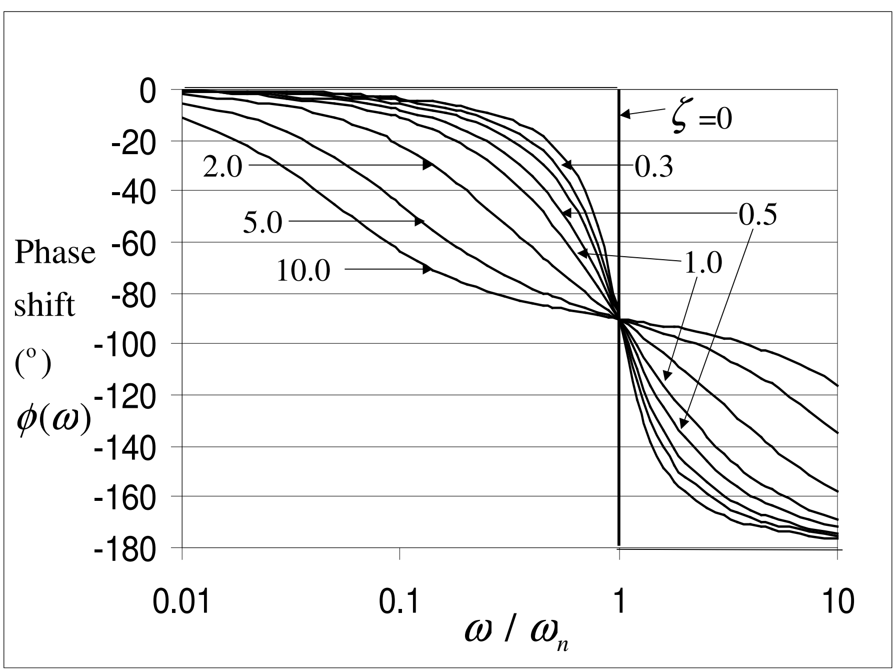
$$y_{steady}(t) = B(\omega)\sin[\omega t + \phi(\omega)]$$

$$M(\omega) = \frac{B}{KA} = \frac{1}{\left\{\left[1-(\omega/\omega_n)^2\right]^2 + (2\zeta\omega/\omega_n)^2\right\}^{1/2}}$$

Magnitude ratio




10
Magnitude
ratio
1
M (ω)
0.1
0.01
0.1
ω / ωn
1
10


0
-20
-40
-60
-80
-100
-120
-140
-160
-180
ζ =0
2.0
0.3
5.0
0.5
10.0
1.0
Phase
shift
(°)
φ(ω)
0.01
0.1
1
10
ω / ωn