System Reliability

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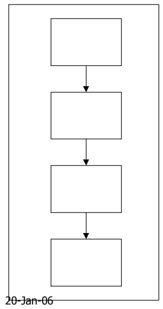


- n parts in series
- Failure of any part causes failure of system
- Reliabilities of parts are independent
- $R_s = R_1 R_2 \dots R_n$

$$R_s = \prod_{i=1}^{n} R_i$$

Problem:

- A communications system has 4 subsystems with reliabilities of 0.970, 0.989, 0.995 and 0.996. What is the system reliability?
- Answer:
- 0.951

If each part has constant failure rate,

$$R_s = e^{-t_1\lambda_1} e^{-t_2\lambda_2} \cdots e^{-t_n\lambda_n}$$

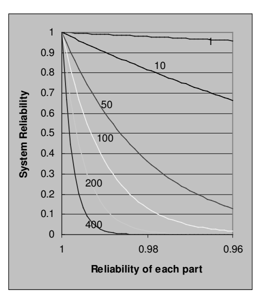
Further, if it is the same for each part,

$$R_s = e^{-t\sum\lambda}$$

Predict the overall failure rate, if all parts must function for system to function

Parts	No of parts	Failure rate, per 10^6 hr	Total failure rate, per 10^6 hr
Resistors	108	0.0048	0.5184
Transistors	23	3.00	69.00
Diodes	50	1.00	50.00
Capacitors	13	0.11	1.43
			120.9484

Effect of increased complexity

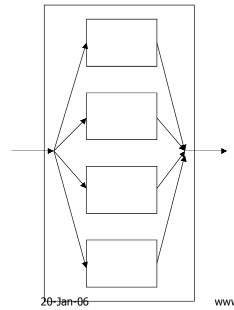


- As the number of parts increases, system reliability decreases dramatically

Redundancy

- Failure of one part will not cause system failure
- Eg:
- Multiple lights in a room
- OHP with extra bulb

Parallel Redundancy



- More than one element for the same job
- Any single element capable of handling the job in case of failure of other elements

n elements in parallel redundancy

- Probability of system failure = Probability that all elements fail at the same time
- If these probabilities are independent,
- $(1-R_s) = (1-R_1)(1-R_2)\ldots(1-R_n)$
- $R_s = 1-[(1-R_1)(1-R_2)\ldots(1-R_n)]$

$$R_s = 1 - \prod_{i=1}^{n}(1-R_i)$$

Problem:

- Suppose a unit has a reliability of 99.0 percent for a specified mission time. If two identical units are used in parallel redundancy, what overall reliability will be obtained?
- R=1-(1-0.99)(1-0.99)
- =0.9999

Problem:

- A factory operates on two identical generators running simultaneously. Each generator is capable of meeting the full load. If the failure rate of the generator is 0.01 per day, determine the system reliability for a period of 30 days.

Solution:

- Reliability of a single generator
- $R_A = e^{-0.01*30} = 0.74082$
- Reliability of system
- R_s= 1-(1-0.74082)(1-0.74082)
- = 0.93282 for a period of 30 days

Note on Failure Rate

- Constant for series systems
- Not constant for parallel systems!
- Even though component failure rates are constant
- Instantaneous failure rate varies as a function of operating time

Comparison of Series and Parallel Failure Rates

	Time	1	2	3	4	5
Component Failure Rate	0.1	0.904837	0.818731	0.740818	0.67032	0.606531
	0.15	0.860708	0.740818	0.637628	0.548812	0.472367
	0.05	0.951229	0.904837	0.860708	0.818731	0.778801
Series Reliability		0.740818	0.548812	0.40657	0.301194	0.22313
Series Failure Rate		0.3	0.3	0.3	0.3	0.3
Parallel Reliability		0.999354	0.995529	0.986918	0.973037	0.954077
Equivalent Parallel Failure rate		0.000647	0.00224	0.00439	0.006833	0.009402

m out of n redundancy

- If out of n identical units actively running in parallel, m units need to be working for the system to function,

R_s = P(0 failures) + P(1 failure) + … + P(i failures) +…+P(n-m failures)

$$R_S = {}^nC_0\left(1-R\right)^0 R^{n-0} + {}^nC_1\left(1-R\right)^1 R^{n-1} + \ldots$$
$$+ {}^nC_i\left(1-R\right)^i R^{n-i} + \ldots + {}^nC_{n-m}\left(1-R\right)^{n-m} R^m$$
$$R_S = \sum_{i=0}^{n-m} {}^nC_i\left(1-R\right)^i R^{n-i}$$

Problem:

- A space vehicle requires 3 out of its 4 main engines to operate in order to achieve orbit. If each engine has a reliability of 0.97, determine the reliability of achieving orbit.

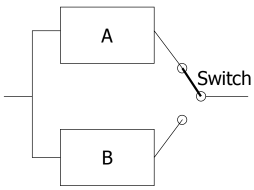
Solution:

$$R_S = \sum_{i=0}^{4-3} {}^4C_i\left(1-R\right)^i R^{4-i}$$
$$R_S = {}^4C_0\left(1-R\right)^0 R^{4-0} + {}^4C_1\left(1-R\right)^1 R^{4-1}$$
$$R_S = \left(0.03\right)^0 (0.97)^4 + 4(0.03)^1(0.97)^3$$
$$R_S = 0.88529 + 0.10952$$
$$R_S = 0.99481$$

Problem (contd):

- If the main engine requires an 8 minute burn-time, determine the MTTF of the **system**. Assume a constant failure rate of 0.0038074/min for each engine.
- R(8)=$e^{-0.0038074*8}$ = 0.97
- From previous problem, R_s=0.99481
- λ_s=6.504e-4
- $MTTF_s$ = 1537min

Standby Redundancy



- 2 unit standby system
- Unit A operates continuously
- When Unit A fails, a sensing and switching unit starts Unit B

Two unit standby reliability

- Reliability of system =
- Probability that unit A works for the whole period t

  OR
- that the sensing and switching unit does not fail by time t_1
- *AND* unit A fails at some time t_1 prior to t
- *AND* unit B successfully functions for the remainder of the mission

Thus,

- $R_s(t)$
- $= R_A(t) + R_{switch}*[1- R_A(t_1)]*R_B(t-t_1)$

Assuming 100% reliability of switch,

- $R_s(t) = R_A(t) + [1- R_A(t_1)]R_B(t-t_1)$
- The time t_1 can be any value from zero (immediate failure of unit A) to t (no failure of unit A).
- For constant failure rates,

$$R_A(t) = e^{-\lambda_A t}$$

$$\left[1 - R_A(t_1)\right] = \int_{t_1=0}^{t_1=t} \lambda_A e^{-\lambda_A t_1} dt_1$$

$$R_B(t-t_1) = e^{-\lambda_B(t-t_1)}$$

$$R_S = e^{-\lambda_A t} + \left(\int_{t_1=0}^{t_1=t} \lambda_A e^{-\lambda_A t_1} dt_1\right)\left(e^{-\lambda_B(t-t_1)}\right)$$

$$R_S = e^{-\lambda_A t} + \lambda_A e^{-\lambda_B t}\left(\int_{t_1=0}^{t_1=t} e^{-(\lambda_A-\lambda_B)t_1} dt_1\right)$$

$$R_S = e^{-\lambda_A t} + \frac{\lambda_A e^{-\lambda_B t}}{\lambda_A - \lambda_B}\left(1 - e^{-(\lambda_A-\lambda_B)t_1}\right)$$

If the failure rate for both units is the same,

$$R_S(t) = e^{-\lambda t} + (\lambda t)e^{-\lambda t}$$

Ref: Smith, Charles O., “Introduction to Reliability in Design”, McGraw Hill, 1976

Problem:

- A factory has two identical generators. One generator runs continuously and the other is switched on only if the first generator fails. If the failure rate of the generator is 0.01 per day, determine the system reliability for a period of 30 days.

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Solution:

- $R_s(t) = e^{-\lambda t} + (\lambda t)\, e^{-\lambda t}$
- $= e^{-0.01*30} + (0.01*30)\, e^{-0.01*30}$
- = 0.74082+ 0.3*0.74082
- =0.96306
- Compare with 2 generators in parallel,
- for which R_s=0.93282

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Problem (contd…)

- Suppose the probability of successful changeover to the second generator is only 0.9, what will be the reliability of the system?
- $R_s(t) = e^{-\lambda t} + R_{switch}*(\lambda t)\, e^{-\lambda t}$
- =0.74082+ 0.9*0.3*0.74082
- =0.94084

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But secondary system can also fail in standby mode,

- If λ^+ is the failure rate of the secondary unit while in standby
- Then, the second term has to be mutilpied by another factor as

$$R_S(t) = e^{-\lambda t} + \frac{\lambda}{\lambda^+}\left(1 - e^{-\lambda^+ t}\right)e^{-\lambda t}$$

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Problem (contd…)

- If the second generator has a failure rate of 0.001 per day while in standby, determine the reliability of the system.

$$\frac{\lambda}{\lambda^+}\left(1 - e^{-\lambda^+ t}\right) = \frac{0.01}{0.001}\left(1 - e^{-0.001*30}\right) = 0.29554$$

- R_s=0.74082 + 0.9*0.29554*0.74082
- =0.95977

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Challenge Problem:

- An active generator has a failure rate of 0.01 per day. An older standby generator has a failure rate of 0.001 while in standby and a failure rate of 0.10 when online. Determine the system reliability for a planned 30 day use and compute the system MTTF.

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Solution

$$R(t)=e^{-0.01t}+\frac{0.01}{0.01+0.001-0.1}\left[e^{-0.1t}-e^{-0.011t}\right]$$

$$R(30)=0.741-0.11236[0.04978-0.7189]$$

$$MTTF=\frac{1}{0.01}+\frac{0.01}{0.1(0.01+0.001)}=109.09 days$$

Ref: Ebeling, Charles E., "An introduction to Reliability and Maintainability Engineering", McGraw Hill, 1987

Standby redundancy for n equal units

$$R_S(t)=e^{-\lambda t}\sum_{i=0}^{n-1}\frac{(\lambda t)^i}{i!}$$

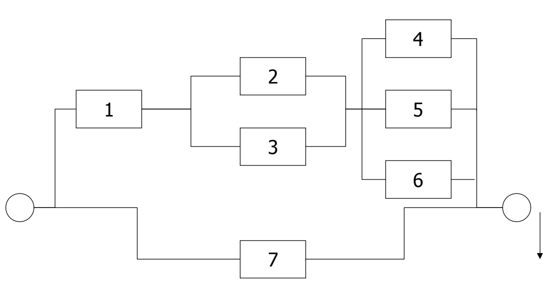
$$R_S(t)=e^{-\lambda t}\left[1+\frac{\lambda t}{1!}+\frac{(\lambda t)^2}{2!}+\ldots\right]$$

Complex Systems

- Convert to the simple cases as above

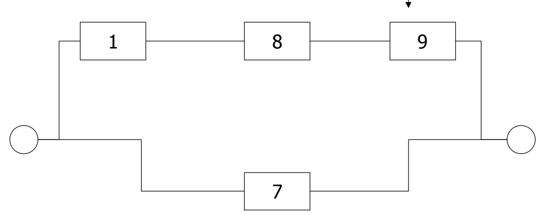
Problem:

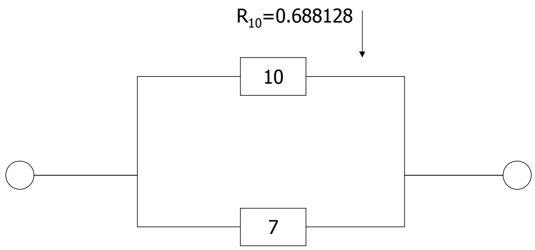
- Determine the reliability of the system if all parallel branches are fully redundant, with the exception of that consisting of components 4, 5 and 6, for which any two of the branches are required for system success
- All components are identical with a reliability of 0.8

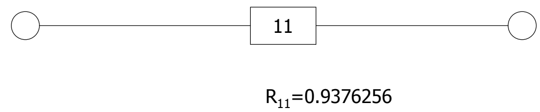


R_8=1- [0.2*0.2]
=0.96

R_9=1- [0.2^3+3*0.8*0.2^2]
=0.8960







Ref: Billinton, Roy and Allan, Ronald N, "Reliability Evaluation of Engineering Systems - Concepts & Techniques", Plenum Press, New York, 1992

Predicting Reliability During Design